Math 124 **Midterm 1 Exam** **Autumn 2016**

Your Name (print) SOLUTION KEY.

Your Signature _____

Student I.D.# | | | | | | | | **Quiz Section** | | |

- Turn in your exam when you are finished. Do not leave the room until the examination is completed. You will lose credit if you leave early.

- Turn off all electronic devices. If your phone rings accidentally, you must ask for permission to turn it off.

- This exam is closed book. You may use one 8.5x11 sheet of handwritten notes (one side), but the notes cannot include completely worked problems.

- The only calculator allowed is the TI-30x IIS.

- In order to receive credit you must show all of your work. Show enough work that the grader can determine what you did to arrive at your answers. Correct answers without justification may not receive much credit.

- If you need more room, use the backs of the pages, but **clearly** indicate you have done so.

Place a box $\boxed{}$ **around your answer whenever that is appropriate.**

Score

1.	(10)	
2.	(30)	
3.	(30)	
4.	(30)	
Total	(100)	

1. Compute the derivative of the following function. Do not simplify your answer.

$$f(x) = \frac{\sin(e^{2x-1}) + 3x - 1}{1 + x\tan(x^3)}$$

$$f'(x) = \frac{[1 + x\tan(x^3)][\cos(e^{2x-1})e^{2x-1} \cdot 2 + 3] - [\sin(e^{2x-1}) + 3x - 1][\tan(x^3) + x\sec^2(x^3)\}3x^2]}{[1 + x\tan(x^3)]^2}$$

2. You can use limits, but do not use differentiation rules in this problem. Show all of your work to receive credit.

Find the slope of the graph of $y = \tan x$ when $x = \pi/4$. You may use the identity:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

and you may use the limit proved in class:

$$\lim_{x \to 0} \frac{\sin h}{h} = 1.$$

Chordal slope:

$$\frac{\tan(\frac{\pi}{4}+h) - \tan \pi/4}{h} = \frac{\frac{\tan\frac{\pi}{4} + \tan h}{1 - \tan\pi/4 \tan h} - \tan \pi/4}{h}$$

$(\tan\frac{\pi}{4} = 1)$

$$= \frac{\frac{1+\tan h}{1-\tan h} - 1}{h}$$

$$= \frac{2\tan h}{h(1-\tan h)}$$

$$= 2\,\frac{\sin h}{h[\cos h][1-\tan h]}$$

$\longrightarrow 2$ as $h \to 0$

because $\frac{\sin h}{h} \to 1$, $\cos h \to 1$, & $\tan h \to 0$ as $h \to 0$

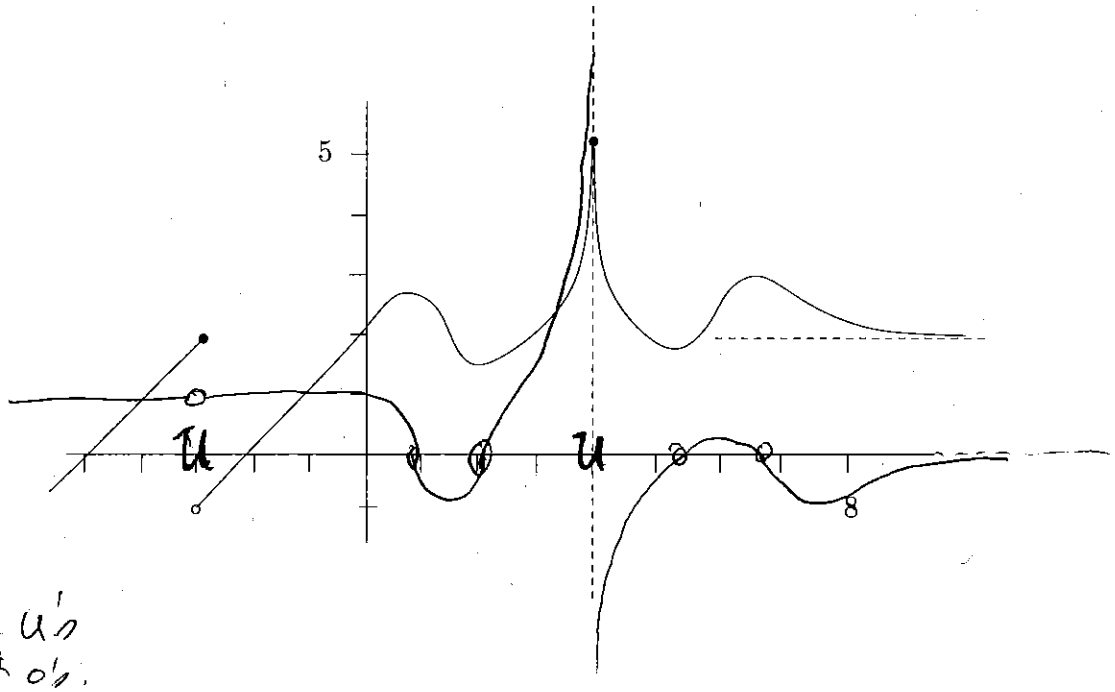
So $\frac{d}{dx}\tan x$ at $x = \pi/4$ is equal to 2.

↑

Slope of the graph when $x = \pi/4$

3. Below is the graph of a function $y = f(x)$.
(a) Put an "0" at all points on the horizontal x-axis (not on the graph) where $f'(x) = 0$ and a "U" where f' is undefined.

(b) Use the information found in part (a) to roughly draw the graph of $y = f'(x)$ using the **same** axes below (on top of the graph of f). You might want to sketch the graph lightly in pencil until you are sure of your answer, then darken the result. If you make a mistake and cannot erase, then draw another coordinate axes and draw the graphs of both f and f'. If you do this on the back of another page, clearly indicate where to find your solution.



2 U's
4 0's.

Each tic mark is one unit on the x and y axis. Find:
(c) $\lim_{x\to 4^-} f'(x) = +\infty$

(d) $\lim_{x\to 4^+} f'(x) = -\infty$

(e) $\lim_{x\to+\infty} f'(x) = 0$

(f) $\lim_{x\to-\infty} f'(x) = 1$

(g) Give the equations of the horizontal asymptotes to the graph $y = f'(x)$ (Note: we are not asking for horizontal asymptotes to the graph of $y = f(x)$.)

$y=0$ and $y=1$

(h) Give the equation of the vertical asymptote to the graph $y = f'(x)$.

$x=4$

4. In this problem you will find an approximate value for x where $\tan(x) - x = \frac{1}{4}$. You might find it helpful to use your calculator, but if you use numbers from your calculator then give at least 6 digits after the decimal place. Write enough details so that the grader can tell exactly what you did on your calculator.

Let $f(x) = \tan x - x$.

a. Find the equation of the line tangent to the graph of $y = f(x)$ when $x = \pi/4$.

$f(\frac{\pi}{4}) = 1 - \pi/4 \quad f'(x) = \sec^2 x - 1 \quad f'(\pi/4) = 2 - 1 = 1$

$y = (x - \pi/4) + 1 - \pi/4 = x + 1 - \pi/2$

b. Where is the linear function found in part a equal to $\frac{1}{4}$? (call the solution x_1).

$x_1 + 1 - \pi/2 = 1/4$

$x_1 = \pi/2 - 3/4 \approx 0.8207963$

x_1

c. Find the equation of the line tangent to the graph of $y = f(x)$ when $x = x_1$. Use the explicit value of x_1 found in part b.

this means "approximately equal to"

$f(x_1) = \tan(.8207963) - 0.8207963 \approx 0.2526298\ldots$

$f'(x_1) = \sec^2(0.8207963) - 1 \approx 1.1522436$

$y = 1.1522436(x - .8207963) + 0.2526298$

d. Where is linear function found in part c equal to $\frac{1}{4}$? (call the solution x_2).

$1.1522436(x_2 - .8207963) + 0.2526298 = 1/4$

$x_2 = .8207963 + \frac{0.25 - 0.2526298}{1.1522436} \approx 0.8185139\ldots$

e. Find $f(x_2)$. (hopefully it is near $\frac{1}{4}$).

$f(x_2) \approx \tan(0.8185139) - 0.8185139 \approx 0.250012\ldots$

very close to 0.25 ✓ -5